# Meanings of physics mathematization in pre-service physics teachers ${ }^{1}$ 

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#### Abstract

Introduction. Physics teaching practice is permeated by the meaning that teachers constructed about the relationship between mathematics and physics, but this relationship often goes unnoticed by the teacher himself, because of their education where it is traditionally thoughtlessly assumed that "mathematics is the physics language." Objective. In this work, we contribute to the comprehension of how modifying this reality in teacher training. To this end, we conducted a study on how is understanding physics and mathematics relationships by preservice teachers. Materials and methods. Data collection was done from observation of a nonparticipating classroom in two physics courses using an observation grid and a questionnaire. We made a content analysis. Results. We found that pre-service teachers consider the mastery of problem-solving equations as the primary medium for learning physics, but when they are asked to explain physics they usually opt for qualitative descriptions and pictorial representations not for equations. Conclusions. It means that they


differentiate between a "mathematical part" of the phenomenon that is self-controlled as the basis of his physics learning and a "qualitative part" of the phenomenon that serves to explain physics. So, it seems that to teach physics they feel the need to explain conceptually without mathematics, while to learn physics they should concentrate on applying equations, which is paradoxical.

Keywords: Physics Teaching, Mathematical and physics relationship; Physics mathematization.

## Significados de matematización de la física en docentes de física en formación

## Resumen

Introducción. El tipo de trabajo docente en la enseñanza de la física está influenciado por el significado que cada docente ha construido sobre la relación entre las matemáticas y

[^0]la física, pero esta relación a menudo pasa desapercibida para el maestro mismo, ya que es tradición en su formación suponer irreflexivamente que "las matemáticas son el lenguaje de la física". Objetivo. En este trabajo, contribuimos a la comprensión de cómo modificar esta realidad en la formación del profesorado. Con este fin, llevamos a cabo un estudio sobre cómo es la comprensión de las relaciones de física y matemáticas por parte de los docentes en formación. Materiales y métodos. La recolección de datos se realizó a partir de la observación no participante en dos cursos de física utilizando una rejilla de observación y un cuestionario. El analisis se hizo a partir del análisis de contenido. Resultados. Descubrimos que los maestros en formación consideran el dominio de las ecuaciones en la resolución de problemas como el eje fundamental para aprender física, pero cuando se les pide que expliquen física, optan por descripciones cualitativas y representaciones pictóricas y no por ecuaciones. Conclusiones. Los docentes en formación diferencian entre la "parte matemática" del fenómeno al que atribuye la base de su aprendizaje de física y la "parte cualitativa" del fenómeno que sirve para explicar la física. Entonces, parece que para enseñar física sienten la necesidad de explicar conceptualmente sin matemáticas, mientras que para aprender física se concentransolamente en aplicar ecuaciones, lo cual resulta paradójico.

Palabras clave: Enseñanza de la física, relación entre matematica y física, matematización de la física. (deben ser tomadas de un tesauro especializado del tema)

# Significados da matemátização da física em professores de física en formação 

## Resumo

Introdução: O tipo de trabalho docente no ensino de física é influenciado pelo significado que cada professor construiu sobre a relação entre matemática e física, mas essa relação geralmente passa despercebida pelo próprio professor, uma vez que é uma tradição em sua formaçao supõe, sem pensar, que "a matemática é a linguagem da física". Objetivo. Neste trabalho, contribuímos para a compreensão de como modificar essa realidade na formação de professores. Para tanto, realizamos um estudo sobre como é a comprensao das relaçoes entre a física e a matemática na formação de professores de física. Materiais e métodos. A coleta de dados foi realizada a partir da observação mais não do participante em diferentes aulas de física, utilizando um roteiro de observação e um questionário. A análise foi feita a partir da análise de conteúdo. Resultados. Descobrimos que os professores em formação consideram o domínio das equações na solução de problemas como o eixo fundamental para aprender física, mas, quando solicitados a explicar física, optam por descrições qualitativas e representações pictóricas, e não por equações. Conclusões. Os professores em formação diferenciam entre a "parte matemática" do fenômeno à qual ele atribui a base de seu aprendizado da física e a "parte qualitativa" do fenômeno que serve para explicar a física. Parece que, para ensinar física, eles sentem a necessidade de explicar conceitualmente sem matemática, enquanto que para aprender física, eles se concentram apenas na aplicação de equações, o que é paradoxal.

Palavras-chave: Ensino de física, relação entre matemática efísica, matemátização da física.

## Introduction

Authors such as Paty (2003), Gingras, and Cantor (1977), proposed that mathematization of physics has been understood in different ways throughout its history and each one has made its classification in stages. These stages talk about the relationship between mathematics and physics and the epistemological role they play in the development of physics.

Historically, we can observe how mathematization has allowed physics to formalize highly predictive and successful theories (Vizcaino \& Terrazzan, 2015), giving rise to the idea that physics teaching based on algorithmic development will, in turn, be successful.

We consider important to know what meanings are presented by undergraduate students about the relationship between physics and mathematics, because, in initial education teachers is when they consolidate their epistemological notion of how science is done and how it is teaching. At this time, they theoretically develop the bases of their performance in the classroom, plus that some studies like Vizcaino (2013) and Castiblanco, Nardi $(2016,2018)$, indicate how future teachers are interested in improving their teaching practice but do not know how to do so.

## Theoretical framework

The importance that mathematics has had in physics historical development is undeniable according to Bochner (1966) and Holton; Brush (2013), who explains at length the role of mathematics in the development of science. However, to assume physics as merely a set of equations can lead to a restricted vision about learning and teaching physics. For example, focusing physics teaching only on algorithmic
development, without taking in mind that this is just an aspect of learning that is strongly complemented by conceptual development and thinking skills for science, in the sense of Redish; Gupta (2010). Also, in recent studies (Arenas, 2019), we found that to think about mathematization for physics teaching must have a different character from mathematization for physics constructing.

From teaching, we must encourage and allow the student to acquire structuring abilities, at the same time that he gains technical skills and construct scientific knowledge, according to Karam; Pietrocola (2009). While, in the process of physics development, such relationship is built as a set of individual and collective actions of different people in different knowledge areas, that are leading to the consolidation of one phenomenon representation to be studied by specific communities.

Another consideration to be taken into account in teaching is the fact that the way in which pre-service teachers learned physics, is constituted in the methodology they adopt, as shown by Adelantado; Aleixandre; Pérez (1992) and Castiblanco; Vizcaino (2018), even if this student has had in his training abundant disciplines of didactics of science, which is a barrier to breaking out of the tradition of assuming physics teaching from alternative perspectives.

For this reason, we believe it is necessary to establish the relationship between mathematics and physics in teacher training programs, as a research object. We consider that science epistemology around mathematization of physics also plays an essential role in their way of organizing academicals practices and their professional performance.

## Materials and methods

We did a study with pre-service physics teachers from a public university in the province of Sao Paulo state, Brazil. We chose two physics courses of the career, in the semester one in a subject named classical mechanics, and in 7th semester in the subject named modern physics, which we accompanied during one semester in a non-participating observation, and at the end, we applied a questionnaire. To present data we will call the first-semester group as (A) and the seventh-semester group as (B).

To take data, we design an observation grid with the purpose of characterizing meanings by students, to know their performances in front of aspects like 1 . Do they ask questions? 2. Do they draw on the board? 3. Do they answer the teacher's questions orally? 4. Do they answer the issues of the classmates? 5. Do they make analogies and comparisons with knowledge learned in other contexts? 6. Do they take experimental data? 7. Do they analyze experimental data? 8. Do they interpret results obtained in problem-solving?

The final questionnaire included four items concerning classical mechanics topics like 1 . Describe the process you usually use to solve a physics problem. 2. Choose a wave phenomenon, name it and represent it. 3. One way of expressing the Archimedes principle is "the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces and acts in the upward direction at the centre of mass of the displaced fluid"-how would you explain this phenomenon to another person? 4. For you, what is the meaning of the mathematical expression $F=m a$ ?

We made a Pilot questionnaire version with their respective corrections, and after we applied
with students A and B. Data obtained from observation were compared with data obtained from the questionnaire, therefore we present the results around each item of the questionnaire and its relationship with observation, making interpretations about how students assume the relationship between physics and mathematics.

## Results

We consider important to start talking about that the result in front of the questions we proposed for class observation, was basically an absence of these aspects.

The first indicator we were looking for was about whether they asked the teacher questions spontaneously, which we found that no, the questions that eventually arise are of a technical nature but not related to the subject under study. The second aspect was about representations or drawings that students make on the board in the middle of class explanations, the result was null, since there was no opportunity for students to express themselves in public.

The third, was the answers that students give to the teacher, in general, there were no questions from the teacher to the students related to the subject under study, because usually the teacher is explaining and the questions he asks himself are answered, or written questions are asked in the assessments or laboratory guides that students answer in writing or in presentations previously prepared. The fourth aspect was about whether student's dialogues with his classmates on the subject, where we find that they discuss among themselves but mainly to make sure of what the teacher says but not debating or analyzing any concept.

In relation to whether they make analogies or comparisons with the knowledge learned, it was not possible to verify it, since they were
not expressed in public, in this aspect, we observed that in the expositions, for example, about laboratory reports, they use physics applications to highlight the importance of the subject, however, without going deeper into the details of this applications, but only mentioning artefacts names or technologies.

About taking data in the laboratory with analysis of these data and the respective interpretation, we observe that they do it exactly according to the guidance provided by the teacher for the case, but at no time do they question the procedure or take additional data to corroborate or counteract, basically they were dedicated to using the equation indicated by the teacher to replace data obtained and make the graphs requested.

For this reason, we chose to present the following results based on the answers given by students to the questionnaire that we applied at the end of the semester.

## Ideas about using mathematics in physics

In this regard, we compared the answers to the question that asked them about the meaning of the expression $F=m a$, and the answer asking to choose a wavy phenomenon, name it and represent it, along with the evidence in class observation regarding the language they use to present their explanations at times when the teacher asked for it or when they voluntarily participated in the class.

Basically we find two types of trends, one in which the meaning of the equation is the description with words of the magnitudes involved in the mathematical relationship that expresses the equation, and the other in which the magnitudes are named with their respective proportionality relationship, but the response is broadened by trying to talk about the nature of the magnitudes. The first is mainly present
in Group A and the second is mainly present in Group B.

We interpret that the vision of the equation meaning is maintained in the two groups, with the difference that in students of group A, the language is limited to the magnitudes themselves, while students of group B have acquired a more refined language about the magnitudes that allows them to expand their response. However, neither of the two cases shows any intention to talk about, for example, the idealization of physic system that allow formulating such an equation, or the meaning within a paradigm of physics, neither dimensional analysis of measurement units.
$67 \%$ of group A and $20 \%$ of group B were in the first response type, for instance, responded by naming symbols and describing in words the mathematical relationship exposed in the equation. Considering that the question investigates the meaning of mathematical expression, the central significance obtained is to assume it as a tool for calculating values, since they emphasize the proportionality of the magnitudes involved, it means that equation significance is the equation itself, without considering "mathematical processes such as extrapolations and conscious schematizations" in the perspective of Krygovska (1968), which goes beyond the instrumental mastery of mathematical representations as mentioned by Redish \& Gupta (2010).

The central tendency is to describe mathematical equations without referring to its specificities, or another kind of representation, such as graphics, schemes, applications, etc. For example, in movement laws, one can imagine relations between force and mass beyond a proportionality because it is necessary to imagine phenomena occurring in space and time, like the principle of inertia or the actionreaction law. Neither the nature of the concepts
involved, as in the case of force concept would imply speaking about "interactions" evidencing the existence of a force and its vector character.

In the case of acceleration concept, the fact that it does not respond directly to intuitive experiences but to abstract organizations is disregarded. Nor do they reflect on whether the value of the acceleration calculated with the equation is instantaneous, or average, or whether it is produced by a force acting in an instant or during the whole movement.

In general, we observe the absence of phenomenon descriptions based on qualitative aspects or experimental supports, meaning that the way in which these students relate physics and mathematics is assuming mathematical representations as the unique language to represent physical phenomena, without them having enough clarity about how formalisms are interpreted to understand the phenomenon involved.

In the second type of response, where the description of the equation is broader, we find $16 \%$ of the students in group A and $80 \%$ of the group B. Firstly, we find it striking that not all of the students in group A or all of the students in group B have the same capacity to interpret equations, that is, there are some students of the first semester who manages to delve deeper into their descriptions and some of the students of the seventh semester who do not succeed in exceeding the mere description. But most of the students with advanced studies have better mathematical lexicon which allows them to express the equation in a differential way.

Table 1. Representative examples of a type of answers to the question about the meaning of the expression $F=m a$. This is the type of response for most of group A and most of group $B$

| Group A | " $F=m$. a, the well-known expression of Newton's second law tells us that the resulting force |
| ---: | :--- |
| on a particle with mass $m$ is directly proportional to the acceleration. It is a general expression |  |
| that acquires its full physical meaning when we replace $F$ with the sum of the forces applied to |  |
| the body." |  |
|  | - "This mathematical expression shows that force is a vector and has the same sense and direction |
| of acceleration since it is another vector magnitude. Moreover, for the same force applied to |  |
| bodies of different mass, the reaction, that is, the acceleration suffered will be different." |  |

We observe how students in group B refer to the equation as "Newton's second law" and describe it in a wider sense. They talk about aspects such as vector nature, the relationship between force and acceleration from variations
of the amount of movement with constant mass, different accelerations by applying the same force to different masses, types of force such as gravity and weight, and the force applied as a sum of forces.

They talk too, about conditions of the system in which the equation could be used in problems with inertial referential, or in solving problems about projectile-throw. Therefore, for these students describing this equation implies to explain the relationships expressed in itself but there are no explicit ideas about the nature of the concepts of force, mass, and acceleration or specifications about the causes and consequences of this law.

This result contrasts with what was found in the second item, "choose a wave phenomenon, name it and represent it," we can say that in this case mathematical representation was almost non-existent and the main type of representation used was pictorial. Answers were grouped into five types of content on wave phenomena
chosen by the students: 1 - Those that refer to phenomena related to mechanical waves 2Those that involve electromagnetic waves; 3Those that do not speak of a wave phenomenon, but do refer to properties waves characteristics; 4- those that refer to oscillatory behaviours; 5those that do not answer or express ideas without answering the proposed question.

We notice that when an equation is presented to them, they assume it as a law, but when asked to describe a law they do not necessarily resort to an equation but primary to pictorial representations.

Table 2. Classification of wave phenomena choose by group A and group B

| Phenomena | \% Est. | Group $\boldsymbol{A}$ |
| :--- | :---: | :--- |
| Mechanical waves | $52 \%$ | "Wave interference on a string," Sound," Sound waves," Voice is a <br> wave phenomenon, sonorous." |
| Electromagnetic waves | 16 | "The transmission of energy through radio waves," television signal," <br> Apulse of light in the laboratory," microwave." |
| Wave properties | 23 | "Diffraction", "frequency", "It's refraction of light (...)",","Wave <br> superimposition effect (...)", Electromagnetic interference?"" |
| Oscillatory behavior | 3 | "Alternating current. (...)" |
| Doesn't answer the question | 6 | "Internal circuit" or no response. |
| Group B | 30 | "bucket of water," vibrating rope." |
| Mechanical waves | 40 | "Propagation of light. (...)"," electromagnetic radiation emitted by a <br> monoatomic gas,"(...) or wavy nature of light". |
| Electromagnetic waves | 10 | "Drawing ofa hanging spring,", Drawing of a sinusoidal function without <br> name or explanation." |
| Oscillatory behavior | 20 | They related Wave phenomena throughout nature (...)"," A wave <br> phenomena can be identified by the cyclical variation of magnitude <br> and its propagation (...)". |
| Doesn’t answer the question |  |  |

In both groups, approximately $70 \%$ of students refer to phenomena involving mechanical and electromagnetic waves. In mechanical waves, they mentioned mainly the sound. $30 \%$ of students did not talk about wave properties, or oscillatory behavior without naming a particular wave phenomenon but
mentioning properties such as diffraction, reflection or superposition. Group B use more accurate language to describe the phenomenon, but when representing the phenomenon, they mostly use pictorial representations using generalized drawings of a wave image.

In some of the representations of group A, axes of a Cartesian plane were included, but without the intention of representing the relationship between two specific variables and without using scale units of measure, that is, used without taking into account the meaning of graphically representing wave behavior. Other representations used was a generalized pictorial image or icon about the propagation of
sound, without considering propagation in all directions. Such images are not accompanied by equations, conceptualizations, explanations that describe the nature of the phenomenon, or variables that intervene in the observed physical system. Let's look some image examples.

Figure 1. Representations of group A.
Drawings Group A $\quad$ Translation


Group A drawings of students

Figure 2. Representations of group B.
Drawings Group B $\quad$ Translation

$$
\text { D.fracoo. "dizpuríó" do and par una panda } \quad \begin{gathered}
\text { diffraction wave dispersion } \\
\text { through a slot }
\end{gathered}
$$

## Difraçs de ondas:


wave diffraction
wavefront
Group B drawings of students

We interpret that students understand mathematization based solely on the fact that equations represent the relationships between variables, but when they must talk about a physics phenomenon does not use mathematization processes.

## Explanation meaning

To delve into the meaning of "explaining" in the students, we analyzed the question about how they would explain to another person the principle of Archimedes. We hope to see, for
example, whether they imagine processes for understanding the phenomenon or abstraction levels in problem-solving, or previous phases to present an equation.

We found that $29 \%$ of the group A and $20 \%$ in the group B agree in explaining just paraphrasing the sentence, without referring to a step-by-step comprehension process. Without including other analysis ways of this principle such as, elaboration and interpretation of an experiment, or analyzing mathematical formalisms that represent the state of a body that
floats in equilibrium. In this way, we interpret that for these students an "explanation" does not include studying variables involved, it is not important to ask about the object and fluid densities, neither about the relationship between force and pressure exerted by the fluid, or different equilibrium conditions of the physical system.

In the meantime, $35 \%$ of group $A$ and $40 \%$ of group B try to explain based on the strategy of broadening the definition of the principle even though they did not achieve a complete explanation.

We found that just 3\% of group A and 30\% of group B proposed "to explain" using drawings and equations. The drawings used are pictorial representations about a body submerged in a liquid to which the symbols representing the acting forces were written on top of it. The use of equations only appears in one student of group $A$ (3\%), and in one of the group B (10\%), with the purpose in both cases of presenting a calculation tool without any additional explanation or justification of why they presented it.

Finally, $16 \%$ of group A and $10 \%$ of group $B$ graders mentioned that they would use the sensory experience to construct the explanation. Let's look these two cases,

Student A: 'I would explain that when a person enters a pool, he feels his body lighter due to the force called buoyant that balances us concerning that force that makes us stay with our feet on the ground."

Student B: "The Archimedes principle is the basis for the floating of ships and other means of maritime locomotion. This principlesays that when an object is submerged in water, a certain amount of volume is displaced. So, a force called buoyant is on the object, so that it rises vertically. Also, there is another force weight of the object that is exercised in the same direction but in
opposite direction. When those two forces are equal, the object floats on the water.

The observation sought to identify aspects such as language used by students, use of algorithms, use of analogies, hypothetical reasoning, use of mathematical symbols, explanation of concepts, description of problems, interpretation of experimental data and analysis of problem-solving.

## Discussion

These results are consistent with what we observe in class, where the students' permanent concern is to be able to replicate the teacher's explanation. The questions that students usually ask teachers are mainly about algebraic or computational aspects of solving an equation, or questions about what the evaluation will deal with, or clarifications about the responsibility they assume in developing a laboratory practice, or the arrangements for delivering papers. At no time did questions arise with the intention of delving deeper into the physical content. Therefore, we understand that the meaning of "explain" consists of describing the phenomenon in literary form or paraphrasing the statement with the presupposition that its presentation already involves an explanation. They do not consider in the experimentation, or in comparison with real systems its conditions to be able to use their ideas, and without associating this law to describe observables, describe causeeffect among the variables, without questioning, debating, arguing or counter-argue.

In general, there is an absence of other communication intentions like analogies, comparisons with knowledge obtained in other contexts or the phenomenon study based on different representation types. Consequently, we can say that, the essential aspects for students in the construction of their explanations and
their physics learning are related to the mastery of equations. They think this fact allow them to express in the best way possible theories or solve the problems proposed by the teacher. This result is far away from the idea of teaching through different representations types proposed by authors such as Angell (2008), in planned and guided work with the use of diversified activities in the classroom (Martins et al, 2020), and from other studies such Coleoni (2001) focused on problem-solving indicating how students can memorize principles laws, abstract schemes, but are not able to give it the right meaning.

The results of the research, on the planned and guided work with the use of diversified activities in the classroom, indicate that there is evidence that students expand their scientific reasoning skills, when exposed to a pluralist methodology.

## Problem-solving processes

To know about how they assume mathematical processes when faced with solving a theoretical physics problem, we study the first item of the questionnaire about to describe the process that usually use to solve a physics problem. As a result, we note that the most common proposed resolution process is in three or four stages in both groups, which are basically: 1) reading the statement; 2) identification of variables; 3 ) application of equations, and 4) interpretation of results. Those who talk about the latter refer to verifying if the equation solution is correct.

But we note that at least $26 \%$ in group $A$ and $10 \%$ in group B just take two steps to solve a problem, which are to read the statement and apply the equation. And only one student from each group mentions that after finding numerical answer to the problem, one must reflect on its relevance when thinking about it in a real physic system.

According to the class observation, criteria for choosing the right equation is revising into their notebook or searching problems with the same characteristics on the internet or in the textbook, looking to find something similar or in the best of cases, to be able to find the problem solved. We observed that among the students, the solutions available on the internet or in books are very common, which they consult to every time they have to solve a task or prepare a written evaluation.

Neither in the questionnaire nor in the class observation did we find any indicator that teachers and students assumed problems of physics as questions that could help to broaden their understanding of the physical phenomenon, for example, in relation between these problems and historical paradigms, real facts or epistemological questions. This happens because it is presupposed that understanding the equation and its use in problem-solving imply automatically understanding the phenomenon.

## Conclusions

Results showed that the relationship between physics and mathematics in pre-service physics teachers, for this case, is of utilitarian nature, in the sense that it is restricted to the equations mastery to solve problems. This fact contradicts the idea that many authors have shown about "understanding an equation in physics is not limited to connecting symbols with physical variables" Redish; Gupta (2010).

In general, we observed that historical, philosophical and epistemological aspects that could result from the explanatory capacity of different phenomenon representations are ignored. At least beyond illustration to students in chronological data or important names, and stories about the formulation of equations. In this aspect (Vizcaíno; Terrazzan, 2013), have
found similar results in other contexts which would allow us to infer that this problem could be common in different countries, but especially there is a distance between physics teaching and research results in physics teaching.

We can conclude that students' viewpoint about the relationship between physics and mathematics is far from throughout history and far from processes of scientific thought. It's not a common intention to go beyond descriptions of equations to understand its essence whereby concepts explanations are linked to the physical sense and organizing interpretations. Their vision is restricted to the format presented in textbooks or standard classes usually offered at universities focused on solving theoretical problems.

This relationship is not understood as an opportunity to build thinking skills that allow them to correlate different types of representation of a physical phenomenon. Students have not been inserted in their speeches explaining the use of analogies, comparisons with knowledge obtained in other contexts, or interdisciplinary view of science.

We also conclude, that didactic methodologies must take into account the importance that students give to algorithmic development, guiding them along a path that involves a total understanding of the phenomenon, for that, it is necessary starting from mathematical development but without excluding other aspects of equal relevance. "It is important that the teacher has mastery over the conceptual structure of the discipline he teaches, as well as the history and epistemology of that discipline" (Wesendonk, F. S., \& Terrazzan, E. A. 2020). Mathematization for the physics teaching requires teachers to construct knowledge about mathematical processes in the physics development.

Many students, moreover, in the moment of materializing their explanations, in general, opt for qualitative descriptions and pictorial representations not for equations as is the case when asked to solve a problem. It means that they differentiate between a "mathematical part" of the phenomenon that is self-controlled and a "qualitative part" of the phenomenon that serves to explain physics to others. So, it seems that to teach physics they feel the need to explain conceptually without mathematics, while to learn physics they should concentrate on applying equations, which is paradoxical.

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